Journal of Applied Mechanics and Technical Physics, Vol. 47, No. 3, pp. 359–365, 2006

NONLINEAR EFFECTS CAUSED BY PULSED PERIODIC SUPPLY OF ENERGY IN THE VICINITY OF A SYMMETRIC AIRFOIL IN A TRANSONIC FLOW

S. M. Aulchenko, V. P. Zamuraev, and A. P. Kalinina

UDC 533.6.011

Changes in the structure of a transonic flow around a symmetric airfoil and a decrease in the wave drag of the latter, depending on the energy-supply period and on localization and shape of the energy-supply zone, are considered by means of the numerical solution of two-dimensional unsteady equations of gas dynamics. Energy addition to the gas ahead of the closing shock wave in an immediate vicinity of the contour in zones extended along the contour is found to significantly reduce the wave drag of the airfoil. The nature of this decrease in drag is clarified. The existence of a limiting frequency of energy supply is found.

Key words: *transonic flow, wave drag, energy supply, Euler equations.*

Introduction. The possibility of controlling the aerodynamic characteristics of airfoils with the use of external local pulsed periodic supply of energy in transonic flight regimes is considered in the paper. It was shown [1–5] that the wave drag coefficient of the airfoil is almost independent of the shape and position of energy-supply zones downstream of the cross section with the maximum thickness of the airfoil if the energy is supplied in compact zones (whose shape is close to squares or is weakly extended). (A similar conclusion follows from [6] for continuous addition of energy in the supersonic part of the flow in a zone extended along the shock wave.) Actually, this result is a consequence of the linear decrease in wave drag as a function of energy supplied.

It should be noted that Yuriev et al. [6] ignored the shock wave caused by energy addition, which is inconsistent with available theoretical and experimental data. An attempt to repeat these calculations did not reproduce the wave drag decrease obtained in [6] (up to 25%). The principal conclusion on preferability of energy addition in zones extended along the shock wave over zones extended along the airfoil contradicts the results obtained in [7].

In contrast to [1–6], nonlinear effects caused by energy supply in the pulsed periodic regime in rather narrow zones aligned along the airfoil were observed in [7]. Energy addition in this manner leads to significant changes in the flow structure and in the distribution of pressure forces acting on the airfoil. Such significant changes in the flow structure caused by comparatively small energy expenses were previously observed only for supersonic flows (see, e.g., [8–12]). The present paper describes a continuation of studying the shock-wave structure of a transonic flow around a symmetric airfoil [4, 5, 7].

Formulation of the Problem. The mathematical model of the flow employs a system of two-dimensional unsteady equations of gas dynamics (Euler equations) in a conservative form for a gas with a constant ratio of specific heats γ. To solve this system numerically, we use a finite-volume scheme diminishing the total variation (TVD reconstruction) in intervals between the instants of energy addition. The fluxes at the cell boundaries are calculated by the method described in [13]. Integration in time is performed by the Runge–Kutta method of the third order. The computational grid in the physical domain is geometrically adapted to the airfoil contour and is refined in the vicinity of the contour; the grid in the canonical region is rectangular; the size of the computational

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Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; aultch@itam.nsc.ru; zamuraev@itam.nsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 47, No. 3, pp. 64–71, May–June, 2006. Original article submitted June 21, 2005; revision submitted August 2, 2005.

TABLE 1

Calculation variant	x_1	x_2	$\Delta S \cdot 10^4$	Δt	$C_x \cdot 10^2$	$\Delta C_x \cdot 10^2$	$\Delta C_x/C_x$, %
1	—	—	—	—	4.588	—	—
2	3.609	3.693	0.839	0.5	3.916	0.672	14.6
3	3.609	3.693	0.839	0.05	3.498	1.090	23.8
4	3.609	3.693	0.839	0.025	3.526	1.062	23.1
5	3.609	3.693	0.839	0.005	$\approx$3.57	$\approx$1.02	22.2
6	3.523	3.693	0.581	0.10	3.249	1.339	29.2
7	3.523	3.693	0.581	0.05	3.060	1.528	33.3

grid is 352×320. In the model considered, pulsed energy supply is performed instantaneously, and the gas density and velocity remain unchanged thereby. The energy density of the gas e in energy-supply zones increases by $\Delta e = \Delta E/\Delta S$ [ΔE is the total energy supplied in one zone (per unit length in the direction perpendicular to the airfoil plane) and ΔS is the area of the zone]. The energy is supplied symmetrically around the airfoil (all linear sizes are normalized to the airfoil-chord length l, the gas-velocity components u and v and the velocity of sound a to a_∞, the density ρ to ρ_0, the pressure p and the total energy of a unit volume of the gas e to $\rho_0 a_\infty^2$, the energy supplied ΔE to the parameter $\rho_0 a_\infty^2 l^2$, and the time t and the energy-supply period Δt to l/a_∞; ρ_0 is determined from the condition $p_\infty = \rho_0 a_\infty^2$, where p_∞ and a_∞ are the dimensional pressure and velocity of sound in the free stream).

The initial distribution of parameters corresponds to a steady flow around the airfoil without energy supply. In the interval from the beginning of energy supply to the moment a periodic solution was obtained, the problem was solved as an unsteady one. The moment of reaching a periodic solution was determined by comparing the mean values of the drag coefficient of the airfoil.

The efficiency of external energy supply can be determined by comparing the decrease in wave drag and the increase in thrust of the flying vehicle in the case the same amount of energy is supplied in the engine. The engine efficiency equals $\eta = Ru_\infty/W$ (R is the thrust force, u_∞ is the flight velocity, and W is the supplied power) [14]. The thrust force in the cruising flight regime equals the drag force. We can easily estimate the energy supplied in the vicinity of the airfoil as

$$\Delta E \leqslant \gamma \Delta C_x \mathrm{M}_\infty^3 \Delta t/(4\eta),$$

where ΔC_x is the decrease in the wave drag coefficient and M_∞ is the free-stream Mach number. For a given supplied energy, this formula yields an estimate for the engine efficiency that makes external energy supply beneficial.

Calculation Results. The results were obtained for a NACA-0012 airfoil in a perfect gas flow ($\gamma = 1.4$) for $\mathrm{M}_\infty = 0.85$ and a zero angle of attack. The varied parameters were the energy-supply period ($\Delta t = 0.005$–0.5), the positions of energy-supply zones, and the sizes of these zones for the period-averaged power of energy supplied into one zone equal to $\Delta E/\Delta t = 0.02$.

The wave drag coefficients C_x as functions of the energy-supply period are listed in Table 1. The energy was supplied in an immediate vicinity of the airfoil (x_1 and x_2 are the coordinates of the left and right boundaries of energy-supply zones; the airfoil is located in the interval of the streamwise coordinates $3 \leqslant x \leqslant 4$). Variant No. 1 implies the absence of energy supply.

An arbitrary discontinuity starts to decay immediately after energy addition. A shock wave propagating in all directions from the airfoil is formed. The gas is spreading from the energy-supply zone. At low frequencies (high values of the period), the flow structure has enough time to partly recover before the next instant of energy supply. As a result, the closing shock wave does not intersect the energy-supply zone, though it is shifted upstream. In variant No. 2 ($\Delta t = 0.5$), it is in the rear part of this zone (Fig. 1). The shock wave is still fairly strong, and the losses in stagnation pressure are significant. The decrease in wave drag in this case is approximately 15%. If the energy-supply frequency is increased by an order of magnitude (variant No. 3), the flow structure does not have enough time to recover during the period, and the closing shock wave is shifted upstream, crosses the energy-supply zone, and stops at the front boundary inside the zone (the position of the closing shock wave remains unchanged within a period, as in other variants with periodic addition of energy). A low-density wake with a low velocity of the gas ($\mathrm{M} \approx 0.1$) is formed downstream of the energy-supply zone in the rear part of the airfoil. A large vortex arises behind the airfoil. Among the variants described in Table 1 for which the area of the energy-supply zone is

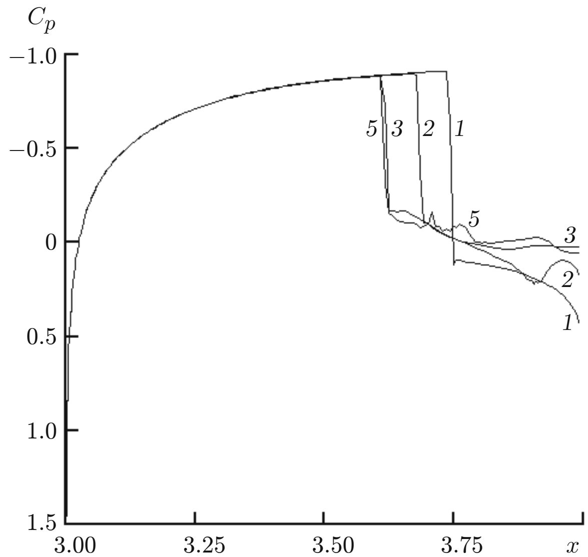

Fig. 1. Distribution of the pressure coefficient C_p along the airfoil chord for different values of the energy-supply period (the numbers of the curves refer to the numbers of calculation variants in Table 1).

TABLE 2

Calculation variant	x_1	x_2	$\Delta y \cdot 10^3$	$\Delta S \cdot 10^4$	$C_x \cdot 10^2$	$\Delta C_x \cdot 10^2$	$\Delta C_x/C_x$, %	η, %
1	—	—	—	—	4.588	—	—	—
2	3.609	3.693	0	0.839	3.498	1.090	23.8	11.7
3	3.567	3.656	0	0.865	3.2432	1.345	29.3	14.5
4	3.523	3.609	0	0.812	2.920	1.668	36.4	17.9
5	3.477	3.567	0	0.830	2.589	1.999	43.6	21.5
6	3.433	3.523	0	0.819	2.250	2.338	51.0	25.1
7	3.352	3.442	0	0.806	≈1.80	≈2.79	60.8	30.0
8	3.271	3.367	0	0.845	≈2.85	≈1.74	37.9	18.7
9	3.433	3.477	0	0.799	2.224	2.364	51.5	25.4
10	3.523	3.693	0	0.581	3.060	1.554	33.7	16.7
11	3.433	3.442	0	0.854	4.116	0.472	10.3	5.1
12	3.609	3.693	0.97	0.839	3.569	1.019	22.2	11.0
13	3.609	3.693	1.94	0.839	3.663	0.925	20.2	9.9

$\Delta S = 0.839 \cdot 10^{-4}$, the greatest decrease in the wave drag coefficient (23.8%) is reached in variant No. 3. With a subsequent increase in energy-supply frequency, the position of the closing shock wave in the periodic solution remains almost unchanged, and the pressure in the rear part of the airfoil decreases. As a consequence, the wave drag coefficient slightly increases. It seems that the periodic solution is not yet stabilized at $\Delta t = 0.005$: the contact discontinuity caused by interaction of the closing shock wave and low-density wake is still unstable. Vortices are formed near the rear part of the airfoil.

In variant Nos. 6 and 7 in Table 1, the length of the energy-supply zone along the airfoil is doubled, and its length in the transverse direction is reduced (more than twofold); hence, the supplied energy density is somewhat higher. In this case, the dependence between the frequency and the decrease in wave drag is retained. The results obtained indicate that there is a limiting frequency of energy supply: for higher values of frequency, the position of the closing shock wave remains unchanged, and no decrease in wave drag is observed with a further increase in frequency. In the considered variants of energy supply in zones extended along the airfoil, the estimated limiting frequency is 20 (the corresponding period of energy supply is 0.05).

Table 2 contains the wave drag coefficients and their decrement at $\Delta t = 0.05$, depending on the positions of the energy-supply zones (Δy is the minimum distance between the zone and the airfoil). In variant No. 1, the

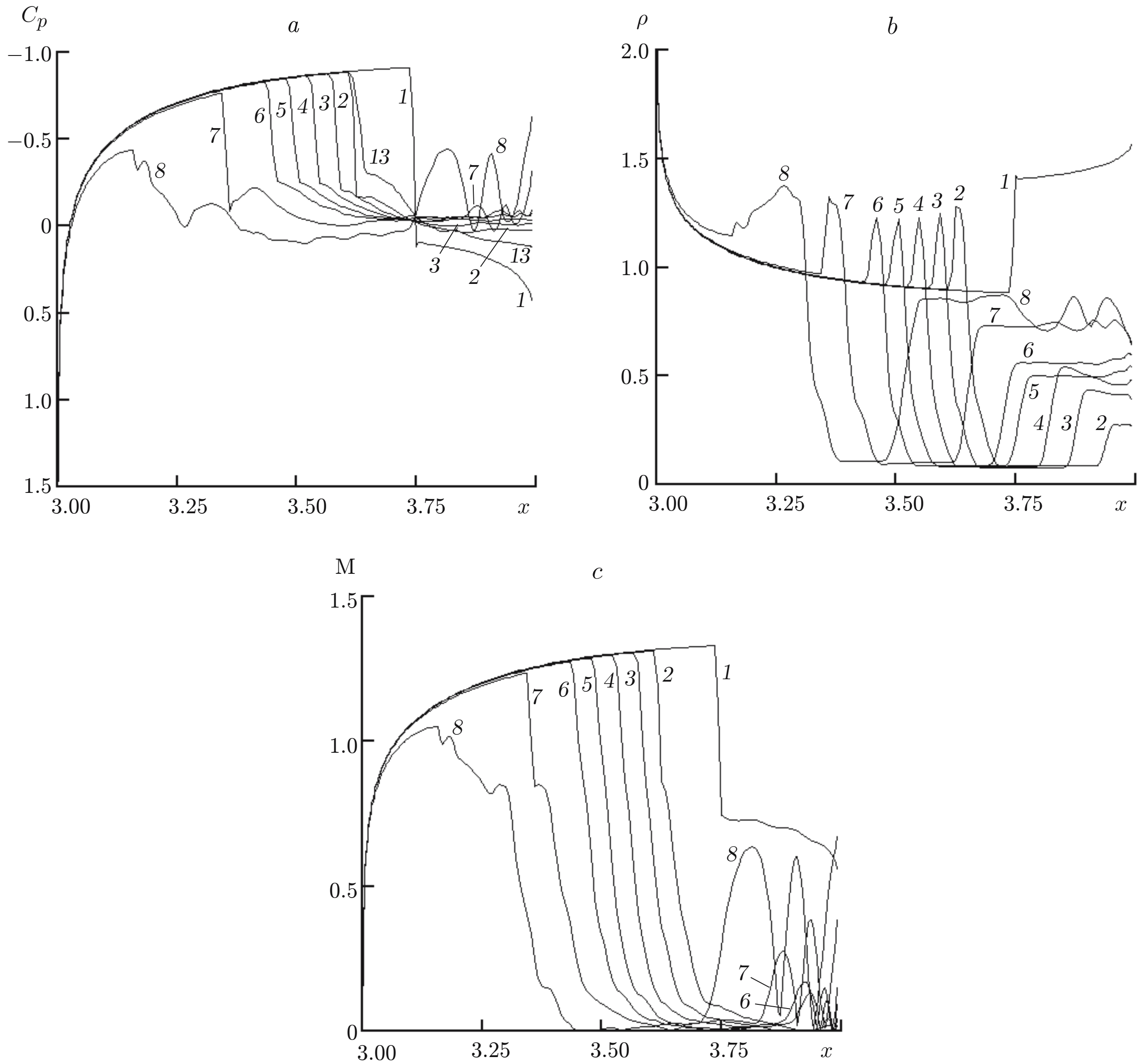

Fig. 2. Distributions of the pressure coefficient (a), gas density (b), and Mach number (c) along the airfoil chord for different positions of the energy-supply zones (the numbers of the curves refer to the numbers of calculation variants in Table 2).

value of the wave drag coefficient corresponds to the absence of energy addition. The last column shows the engine efficiency for which external supply of the considered amount of energy is beneficial. In variant Nos. 2–11, the energy-supply zones are located in an immediate vicinity of the airfoil. It follows from Table 2 that an upstream displacement of the energy-supply zone along the contour (variant Nos. 2–7) up to the mid-section ($x \approx 3.303$) leads to a significant decrease in the wave drag coefficient (the greatest decrease of 60% was obtained in variant No. 7); if the energy-supply zone is shifted to the frontal part of the airfoil, upstream of the mid-section (variant No. 8), the effect of energy supply is not so pronounced. The distributions of the pressure coefficient C_p, gas density ρ, and Mach number M for these variants are plotted in Fig. 2. A significant decrease in wave drag is observed if the closing shock wave is shifted upstream. In variant Nos. 2–7, the closing shock wave is localized in the vicinity of the frontal part of the energy-supply zone. Despite a certain decrease in pressure in the rear part of the airfoil, an increase in pressure on the leeward side owing to closing shock wave displacement reduces the wave drag. In variant No. 8, the supersonic zone is almost completely destroyed (see Fig. 2c), and the pressure on a large part of the airfoil is higher than that in the absence of energy addition.

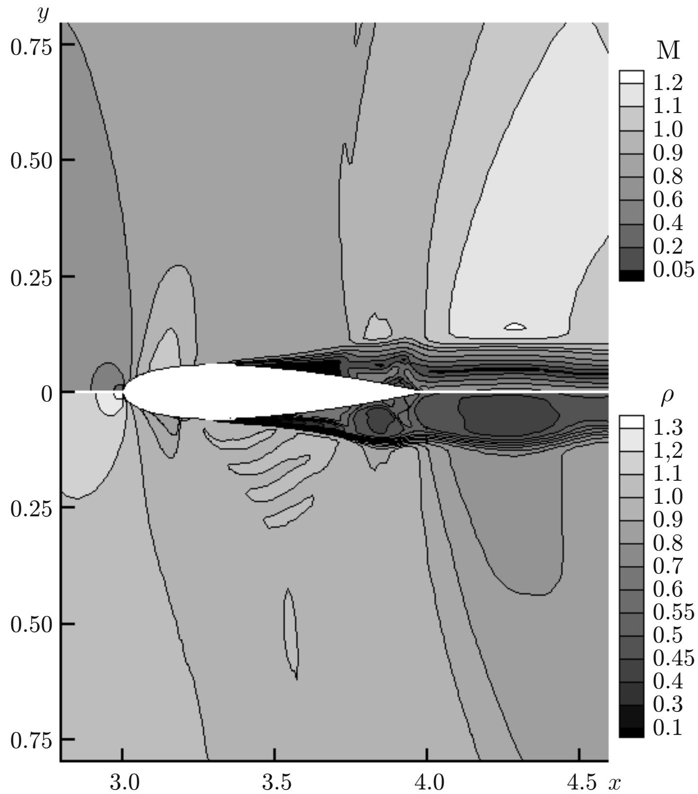

Fig. 3. Isolines of the Mach number (above) and gas density (below) for the case of energy addition in the vicinity of the mid-section.

A low-density wake is formed at a small distance from the closing shock wave (see Fig. 2b). The flow velocity in this wake is low, and the gas density is almost constant and almost identical in all variants under consideration (it is determined by the energy power supplied). The constant-density region decreases as the energy-supply zone is shifted upstream, and vortices arise in the rear part of the airfoil. As flow velocities are rather moderate (about 0.1), the intensity of vortices is low. Nevertheless, it is because of their presence that the periodic solution is not stabilized in variant Nos. 7 and 8. Vortices are the reason for a weaker decrease in wave drag in variant No. 8. The flow above the vortex region is again accelerated to supersonic velocities (therefore, the pressure on the rear part of the airfoil is lower). Figure 3 shows the distributions of the Mach number and density for variant No. 8. Regions with $M > 1.1$ are seen above the wake. Almost complete disintegration of the supersonic region above the airfoil is also observed. The low-density region (wake) is bounded by a contact discontinuity, which is unstable and oscillating (Fig. 3).

A comparison of the values of C_x obtained for variant Nos. 6 and 9 shows that a twofold decrease in length (along the x axis) of the energy-supply zones with an almost unchanged area does not affect the wave drag. In variant Nos. 2 and 10, the length of the energy-supply zones also differs by a factor of 2; their area in variant No. 10 is 1.44 times smaller than that in variant No. 2, whereas the value of $\Delta C_x/C_x$, vice versa, is 1.42 times higher, which can also be related to the upstream displacement of the zone in variant No. 10.

A further decrease in length of the energy-supply zones results in lower efficiency of energy addition, which follows from a comparison of the values of C_x in variant Nos. 6, 9, and 11. In variant No. 11, the length of the energy-supply zones is smaller by an order of magnitude than that in variant No. 6 (the difference in area is approximately 4%), whereas the value of $\Delta C_x/C_x$ is five times smaller than that in variant No. 6.

Table 2 also contain results calculated for the case where the energy-supply zones are shifted by a certain distance Δy from the airfoil. Comparing the values of C_x in variant Nos. 2, 12, and 13, we can see that energy addition in an immediate vicinity of the airfoil is more beneficial: the drag coefficient increases with distance from the

TABLE 3

$t/\Delta t$	$C_x \cdot 10^2$	$t/\Delta t$	$C_x \cdot 10^2$
0.1	1.177	0.6	2.379
0.2	2.137	0.7	2.388
0.3	2.275	0.8	2.392
0.4	2.335	0.9	2.396
0.5	2.364	1.0	2.398

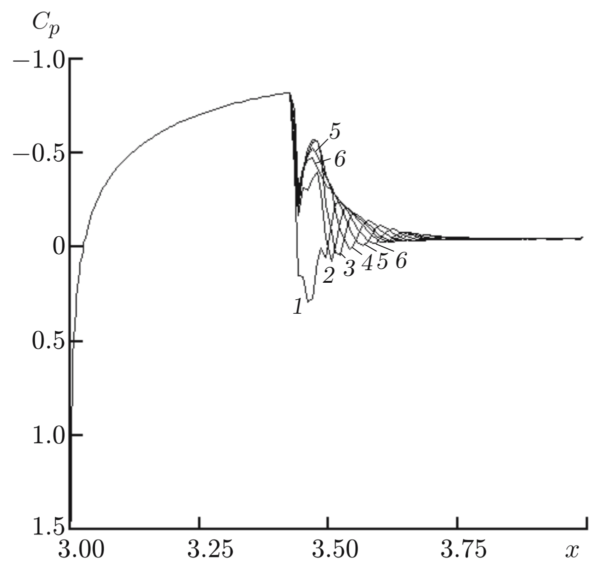

Fig. 4. Distribution of the pressure coefficient along the airfoil chord at different times within one period: $t/\Delta t = 0.1$ (1), 0.2 (2), 0.3 (3), 0.5 (4), 0.7 (5), and 0.9 (6).

airfoil. This behavior of C_x is caused by attenuation of the effect of reflection of disturbances introduced by energy addition from the airfoil with increasing distance between the energy-supply zones and the airfoil. A comparison of curves 2 and 13 in Fig. 2a shows that the closing shock wave is much weaker in the latter case.

The values of C_x listed in Table 2 were obtained by averaging over a time interval equal to 0.5. The dynamics of variation of the wave drag coefficient within one period can be evaluated on the basis of the values of C_x given in Table 3 for variant No. 9 from Table 2. These values of C_x were obtained by averaging over the time interval equal to $0.1\Delta t$. The value averaged over the entire period is $C_x = 0.02224$. The distribution of the wave drag coefficient over the larger part of the period is fairly uniform (the deviations from the mean value over the period do not exceed 5%).

Figure 2 shows the spatial distribution of flow parameters at the moment before energy addition. The distribution of the pressure coefficient over the airfoil at different times for the parameters listed in Table 3 is plotted in Fig. 4. As a result of energy addition, the pressure in the corresponding zones drastically increases, and shock waves are formed and propagate in all directions from the airfoil (curve 1 shows only the front of the wave propagating downstream), with expansion waves developed behind these shock waves. After that, the pressure in the energy-supply zones decreases because of gas spreading (curves 2–4). In what follows, the intensity of the downstream propagating shock wave decreases, and this shock wave is entrained by the flow (curves 5 and 6). The pressure in the energy-supply zone increases, and a distribution preceding the next addition of energy is formed. It is seen from Fig. 4 that the position of the shock wave closing the supersonic region remains almost unchanged during the period. There are regions of almost constant pressure with low gas velocities behind the shock wave.

Conclusions. The results of the present study show that the use of external periodic energy supply aimed at controlling both local and integral characteristics of airfoils in transonic flow regimes are fairly promising. A significant (more than twofold) decrease in wave drag of the airfoil is observed owing to the nonlinear character of interaction of disturbances introduced into the flow by energy addition with the closing shock wave and airfoil surface. In this case, energy supply is beneficial if the engine efficiency is less than 30%. The existence of a limiting frequency of energy addition is established: an further increase in frequency does not reduce the wave drag. The periodic character of the flow being formed allows its use in cruising flight regimes. It offers an opportunity of designing transonic airfoils with the maximum cruising Mach number with satisfied geometric and gas-dynamic restrictions and retained given lift force under conditions of energy addition.

The authors are grateful to P. Yu. Georgievsky for useful discussions of this problem.

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